

CARNOT CYCLE

The Carnot cycle has the greatest efficiency possible of an engine (although other cycles have the same efficiency) based on the assumption of the absence of incidental wasteful processes such as friction, and the assumption of no conduction of heat between different parts of the engine at different temperatures.

Introduction

In the early 19th century, steam engines came to play an increasingly important role in industry and transportation. However, a systematic set of theories of the conversion of thermal energy to motive power by steam engines had not yet been developed. Nicolas Léonard Sadi Carnot (1796-1832), a French military engineer, published *Reflections on the Motive Power of Fire* in 1824. The book proposed a generalized theory of heat engines, as well as an idealized model of a thermodynamic system for a heat engine that is now known as the Carnot cycle. Carnot developed the foundation of the second law of thermodynamics, and is often described as the "Father of thermodynamics."

The Carnot Cycle

The Carnot cycle consists of the following four processes:

- I. A reversible isothermal gas expansion process. In this process, the ideal gas in the system absorbs q_{in} amount heat from a heat source at a high temperature T_h , expands and does work on surroundings.
- II. A reversible adiabatic gas expansion process. In this process, the system is thermally insulated. The gas continues to expand and do work on surroundings, which causes the system to cool to a lower temperature, T_l .

- III. A reversible isothermal gas compression process. In this process, surroundings do work to the gas at T_l , and causes a loss of heat, q_{out} .
- IV. A reversible adiabatic gas compression process. In this process, the system is thermally insulated. Surroundings continue to do work to the gas, which causes the temperature to rise back to T_h .

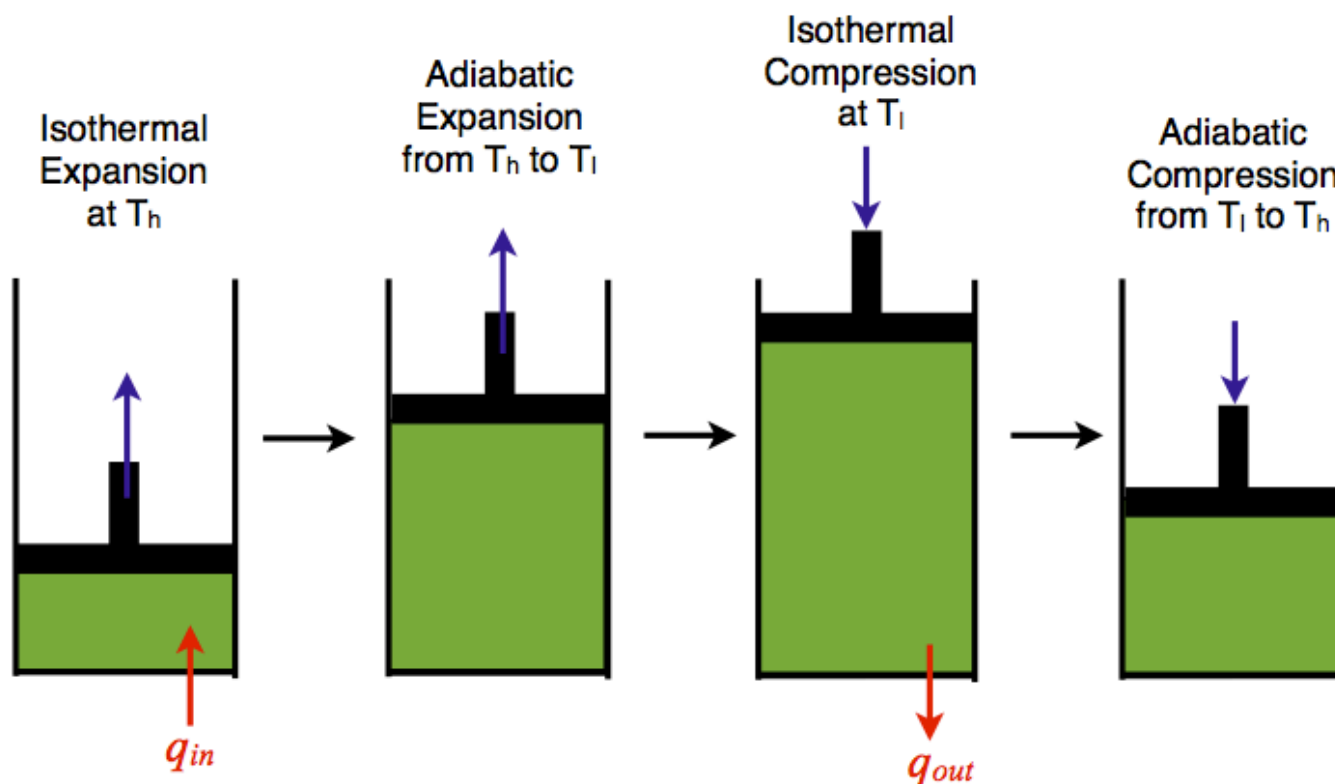


Figure 1: An ideal gas-piston model of the Carnot cycle.

P-V Diagram

The P-V diagram of the Carnot cycle is shown in Figure 2. In isothermal processes I and III, $\Delta U=0$ because $\Delta T=0$. In adiabatic processes II and IV, $q=0$. Work, heat, ΔU , and ΔH of each process in the Carnot cycle are summarized in Table 1.

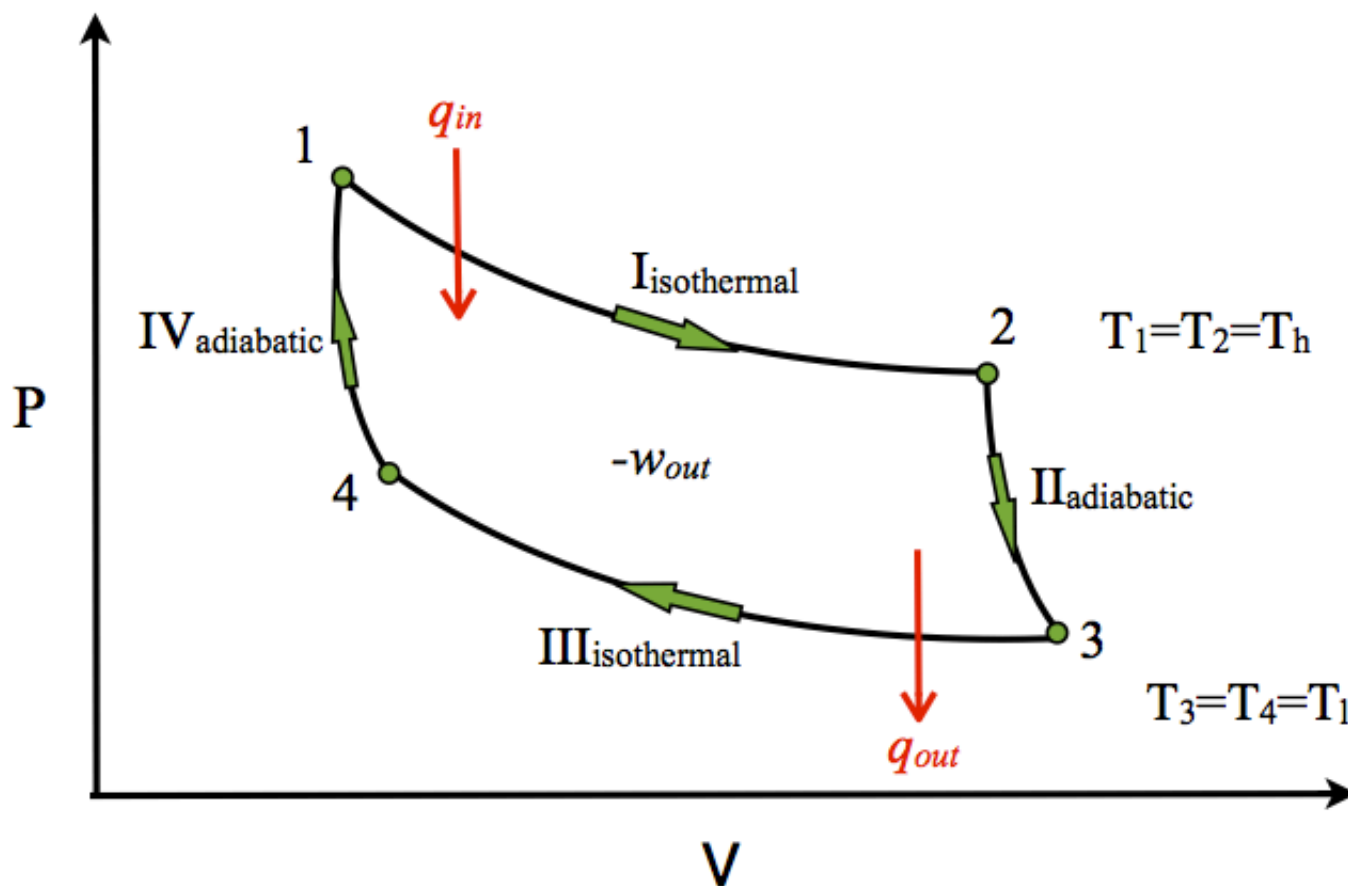


Figure 2: A P-V diagram of the Carnot Cycle.

Table 1. Work, heat, ΔU , and ΔH in the P-V diagram of the Carnot Cycle.

| Process | w | q | ΔU | ΔH |
|------------|--|---|-------------------------|-------------------------|
| I | $-nRT_h \ln\left(\frac{V_2}{V_1}\right)$ | $nRT_h \ln\left(\frac{V_2}{V_1}\right)$ | 0 | 0 |
| II | $n\bar{C}_v(T_l - T_h)$ | 0 | $n\bar{C}_v(T_l - T_h)$ | $n\bar{C}_p(T_l - T_h)$ |
| III | $-nRT_l \ln\left(\frac{V_4}{V_3}\right)$ | $nRT_l \ln\left(\frac{V_4}{V_3}\right)$ | 0 | 0 |
| IV | $n\bar{C}_v(T_h - T_l)$ | 0 | $n\bar{C}_v(T_h - T_l)$ | $n\bar{C}_p(T_h - T_l)$ |
| Full Cycle | $-nRT_h \ln\left(\frac{V_2}{V_1}\right) + nRT_l \ln\left(\frac{V_4}{V_3}\right)$ | $nRT_h \ln\left(\frac{V_2}{V_1}\right) - nRT_l \ln\left(\frac{V_4}{V_3}\right)$ | 0 | 0 |

T-S Diagram

The T-S diagram of the Carnot cycle is shown in Figure 3. In isothermal processes I and III, $\Delta T=0$. In adiabatic processes II and IV, $\Delta S=0$ because $dq=0$. ΔT and ΔS of each process in the Carnot cycle are shown in Table 2.

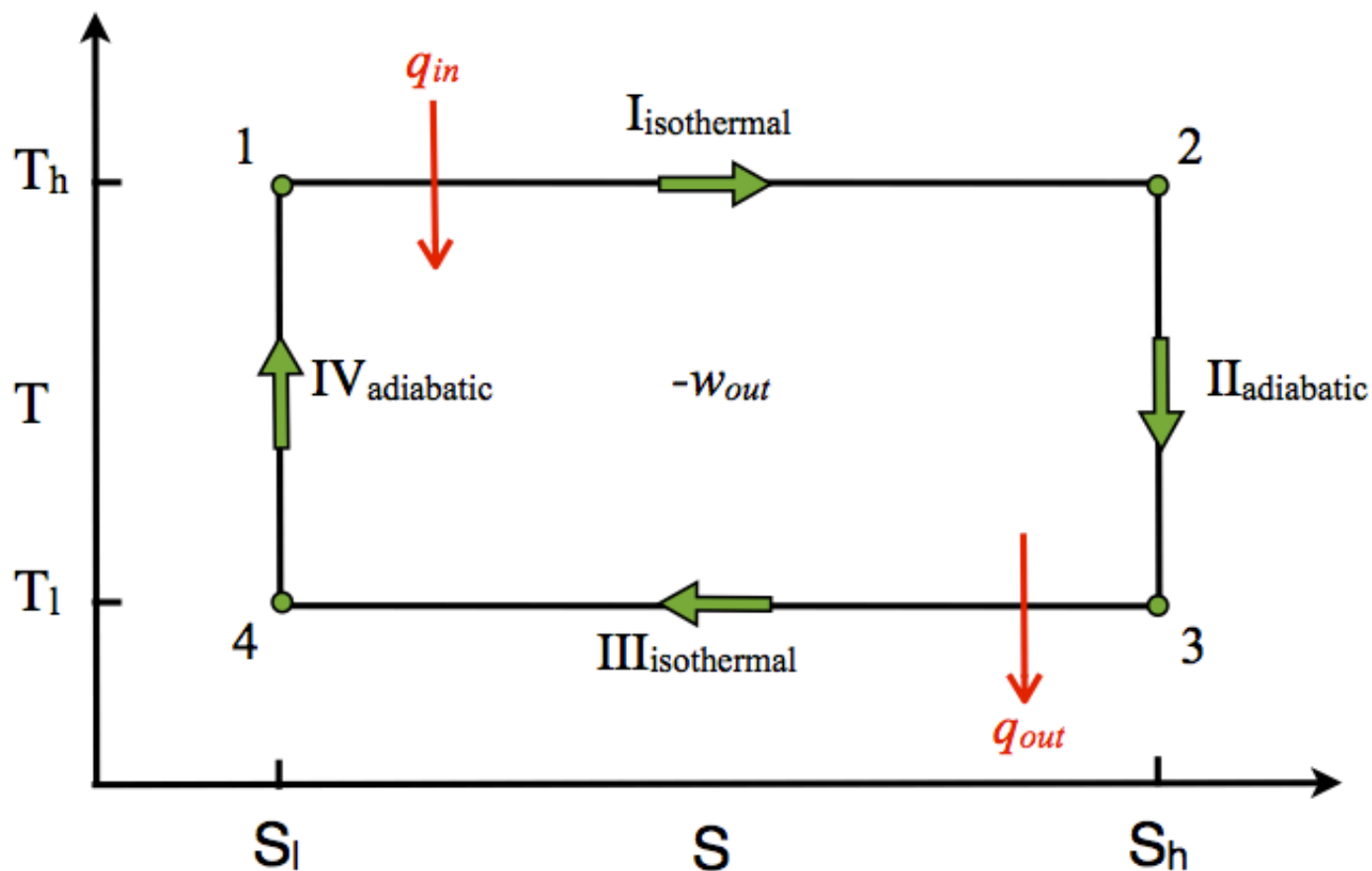


Figure 3: A T-S diagram of the Carnot Cycle.

Table 2. Work, heat, and ΔU in the T-S diagram of the Carnot Cycle.

| Process | ΔT | ΔS |
|---------|-------------|--|
| I | 0 | $-nR \ln \left(\frac{V_2}{V_1} \right)$ |
| II | $T_l - T_h$ | 0 |

| | | |
|------------|-------------|--|
| III | 0 | $-nR \ln \left(\frac{V_4}{V_3} \right)$ |
| IV | $T_h - T_l$ | 0 |
| Full Cycle | 0 | 0 |

Efficiency

The Carnot cycle is the most efficient engine possible based on the assumption of the absence of incidental wasteful processes such as friction, and the assumption of no conduction of heat between different parts of the engine at different temperatures. The efficiency of the Carnot engine is defined as the ratio of the energy output to the energy input.

$$\text{efficiency} = \frac{\text{net work done by heat engine}}{\text{heat absorbed by heat engine}} = \frac{-w_{\text{sys}}}{q_h} \quad (1.1)$$

$$= \frac{nRT_h \ln \left(\frac{V_2}{V_1} \right) + nRT_l \ln \left(\frac{V_4}{V_3} \right)}{nRT_h \ln \left(\frac{V_2}{V_1} \right)} \quad (1.2)$$

Since processes II (2-3) and IV (4-1) are adiabatic,

$$\left(\frac{T_2}{T_3} \right)^{C_v/R} = \frac{V_3}{V_2} \quad (1.3)$$

and

$$\left(\frac{T_1}{T_4} \right)^{C_v/R} = \frac{V_4}{V_1} \quad (1.4)$$

And since $T_1 = T_2$ and $T_3 = T_4$,

$$\frac{V_3}{V_4} = \frac{V_2}{V_1} \quad (1.5)$$

Therefore,

$$\text{efficiency} = \frac{nRT_h \ln\left(\frac{V_2}{V_1}\right) - nRT_l \ln\left(\frac{V_2}{V_1}\right)}{nRT_h \ln\left(\frac{V_2}{V_1}\right)} \quad (1.6)$$

$$\boxed{\text{efficiency} = \frac{T_h - T_l}{T_h}} \quad (1.7)$$

Problems

1. You are now operating a Carnot engine at 40% efficiency, which exhausts heat into a heat sink at 298 K. If you want to increase the efficiency of the engine to 65%, to what temperature would you have to raise the heat reservoir?
2. A Carnot engine absorbed 1.0 kJ of heat at 300 K, and exhausted 400 J of heat at the end of the cycle. What is the temperature at the end of the cycle?
3. An indoor heater operating on the Carnot cycle is warming the house up at a rate of 30 kJ/s to maintain the indoor temperature at 72 °F. What is the power operating the heater if the outdoor temperature is 30 °F?

References

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